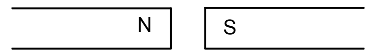
CHAPTER – 36
PERMANENT MAGNETS

1. m = 10 A-m, d = 5 cm = 0.05 m



$$B = \frac{\mu_0}{4\pi}\frac{m}{r^2} = \frac{10^{-7} \times 10}{\left(5 \times 10^{-2}\right)^2} = \frac{10^{-2}}{25} = 4 \times 10^{-4} \text{ Tesla}$$

2. $m_1 = m_2$ = 10 A-m
r = 2 cm = 0.02 m
we know

Force exerted by tow magnetic poles on each other = $\frac{\mu_0}{4\pi}\frac{m_1 m_2}{r^2} = \frac{4\pi \times 10^{-7} \times 10^2}{4\pi \times 4 \times 10^{-4}} = 2.5 \times 10^{-2}$ N

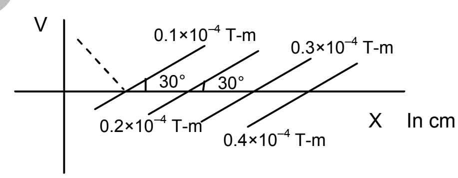
3. $B = -\frac{dv}{d\ell} \Rightarrow dv = -B\,d\ell = -0.2 \times 10^{-3} \times 0.5 = -0.1 \times 10^{-3}$ T-m

Since the sigh is –ve therefore potential decreases.

4. Here dx = 10 sin 30° cm = 5 cm

$$\frac{dV}{dx} = B = \frac{0.1 \times 10^{-4} \text{ T-m}}{5 \times 10^{-2} \text{ m}}$$

Since B is perpendicular to equipotential surface.
Here it is at angle 120° with (+ve) x-axis and B = 2×10^{-4} T



5. B = 2×10^{-4} T
d = 10 cm = 0.1 m
(a) if the point at end-on postion.

$$B = \frac{\mu_0}{4\pi}\frac{2M}{d^3} \Rightarrow 2 \times 10^{-4} = \frac{10^{-7} \times 2M}{(10^{-1})^3}$$

$$\Rightarrow \frac{2 \times 10^{-4} \times 10^{-3}}{10^{-7} \times 2} = M \Rightarrow M = 1 \text{ Am}^2$$

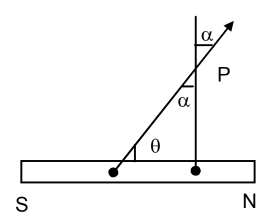
(b) If the point is at broad-on position

$$\frac{\mu_0}{4\pi}\frac{M}{d^3} \Rightarrow 2 \times 10^{-4} = \frac{10^{-7} \times M}{(10^{-1})^3} \Rightarrow M = 2 \text{ Am}^2$$

6. Given :

$$\theta = \tan^{-1}\sqrt{2} \Rightarrow \tan\theta = \sqrt{2} \Rightarrow 2 = \tan^2\theta$$

$$\Rightarrow \tan\theta = 2\cot\theta \Rightarrow \frac{\tan\theta}{2} = \cot\theta$$



We know $\frac{\tan\theta}{2} = \tan\alpha$

Comparing we get, $\tan\alpha = \cot\theta$

or, $\tan\alpha = \tan(90 - \theta)$ or $\alpha = 90 - \theta$ or $\theta + \alpha = 90$

Hence magnetic field due to the dipole is $\perp$r to the magnetic axis.

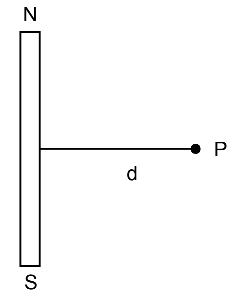
7. Magnetic field at the broad side on position :

$$B = \frac{\mu_0}{4\pi}\frac{M}{\left(d^2 + \ell^2\right)^{3/2}} \qquad 2\ell = 8 \text{ cm} \qquad d = 3 \text{ cm}$$

$$\Rightarrow 4 \times 10^{-6} = \frac{10^{-7} \times m \times 8 \times 10^{-2}}{\left(9 \times 10^{-4} + 16 \times 10^{-4}\right)^{3/2}} \Rightarrow 4 \times 10^{-6} = \frac{10^{-9} \times m \times 8}{\left(10^{-4}\right)^{3/2} + (25)^{3/2}}$$

$$\Rightarrow m = \frac{4 \times 10^{-6} \times 125 \times 10^{-8}}{8 \times 10^{-9}} = 62.5 \times 10^{-5} \text{ A-m}$$

8. We know for a magnetic dipole with its north pointing the north, the neutral point in the broadside on position.



Again $\vec{B}$ in this case = $\frac{\mu_0 M}{4\pi d^3}$

$\therefore \frac{\mu_0 M}{4\pi d^3} = \overrightarrow{B_H}$ due to earth

$\Rightarrow \frac{10^{-7} \times 1.44}{d^3} = 18\ \mu T$

$\Rightarrow \frac{10^{-7} \times 1.44}{d^3} = 18 \times 10^{-6}$

$\Rightarrow d^3 = 8 \times 10^{-3}$

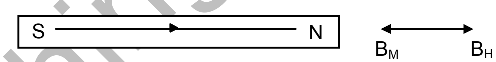
$\Rightarrow d = 2 \times 10^{-1}$ m = 20 cm

In the plane bisecting the dipole.

9. When the magnet is such that its North faces the geographic south of earth. The neutral point lies along the axial line of the magnet.

$\frac{\mu_0}{4\pi}\frac{2M}{d^3} = 18 \times 10^{-6} \Rightarrow \frac{10^{-7} \times 2 \times 0.72}{d^3} = 18 \times 10^{-6} \Rightarrow d^3 = \frac{2 \times 0.7 \times 10^{-7}}{18 \times 10^{-6}}$

$\Rightarrow d = \left(\frac{8 \times 10^{-9}}{10^{-6}}\right)^{1/3} = 2 \times 10^{-1}$ m = 20 cm



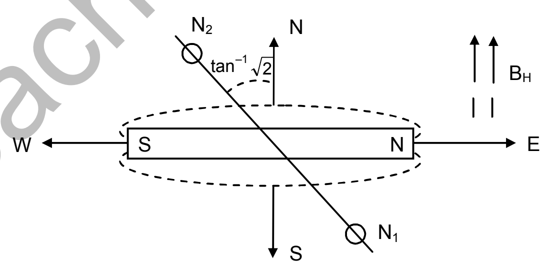
10. Magnetic moment = $0.72\sqrt{2}$ A-m^2 = M

$B = \frac{\mu_0}{4\pi}\frac{M}{d^3}$ $\quad B_H = 18\ \mu T$

$\Rightarrow \frac{4\pi \times 10^{-7} \times 0.72\sqrt{2}}{4\pi \times d^3} = 18 \times 10^{-6}$

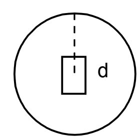
$\Rightarrow d^3 = \frac{0.72 \times 1.414 \times 10^{-7}}{18 \times 10^{-6}} = 0.005656$

$\Rightarrow d \approx 0.2$ m = 20 cm



11. The geomagnetic pole is at the end on position of the earth.

$B = \frac{\mu_0}{4\pi}\frac{2M}{d^3} = \frac{10^{-7} \times 2 \times 8 \times 10^{22}}{(6400 \times 10^3)^3} \approx 60 \times 10^{-6}$ T = 60 μT



12. $\vec{B} = 3.4 \times 10^{-5}$ T

Given $\frac{\mu_0}{4\pi}\frac{M}{R^3} = 3.4 \times 10^{-5}$

$\Rightarrow M = \frac{3.4 \times 10^{-5} \times R^3 \times 4\pi}{4\pi \times 10^{-7}} = 3.4 \times 10^2 R^3$

$\vec{B}$ at Poles = $\frac{\mu_0}{4\pi}\frac{2M}{R^3}$ = = 6.8×10^{-5} T

13. δ(dip) = 60°

$B_H = B \cos 60°$

$\Rightarrow B = 52 \times 10^{-6} = 52\ \mu T$

$B_V = B \sin\delta = 52 \times 10^{-6}\ \frac{\sqrt{3}}{2} = 44.98\ \mu T \approx 45\ \mu T$

14. If δ_1 and δ_2 be the apparent dips shown by the dip circle in the 2$\perp$r positions, the true dip δ is given by

$\text{Cot}^2\delta = \text{Cot}^2\delta_1 + \text{Cot}^2\delta_2$

$\Rightarrow \text{Cot}^2\delta = \text{Cot}^2 45° + \text{Cot}^2 53°$

$\Rightarrow \text{Cot}^2\delta = 1.56 \Rightarrow \delta = 38.6 \approx 39°$

15. We know $B_H = \frac{\mu_0 in}{2r}$

Give : $B_H = 3.6 \times 10^{-5}$ T $\quad \theta = 45°$

$i = 10$ mA $= 10^{-2}$ A $\quad \tan\theta = 1$

$n = ?$ $\quad r = 10$ cm $= 0.1$ m

$$n = \frac{B_H \tan\theta \times 2r}{\mu_0 i} = \frac{3.6 \times 10^{-5} \times 2 \times 1 \times 10^{-1}}{4\pi \times 10^{-7} \times 10^{-2}} = 0.5732 \times 10^3 \approx 573 \text{ turns}$$

16. $n = 50$ $\quad A = 2$ cm × 2 cm = $2 \times 2 \times 10^{-4}$ m^2

$i = 20 \times 10^{-3}$ A $\quad B = 0.5$ T

$$\tau = ni(\vec{A} \times \vec{B}) = niAB \sin 90° = 50 \times 20 \times 10^{-3} \times 4 \times 10^{-4} \times 0.5 = 2 \times 10^{-4} \text{ N-M}$$

17. Given $\theta = 37°$ $\quad d = 10$ cm $= 0.1$ m

We know

$$\frac{M}{B_H} = \frac{4\pi}{\mu_0}\frac{(d^2 - \ell^2)^2}{2d}\tan\theta = \frac{4\pi}{\mu_0} \times \frac{d^4}{2d}\tan\theta \quad \text{[As the magnet is short]}$$

$$= \frac{4\pi}{4\pi \times 10^{-7}} \times \frac{(0.1)^3}{2} \times \tan 37° = 0.5 \times 0.75 \times 1 \times 10^{-3} \times 10^7 = 0.375 \times 10^4 = 3.75 \times 10^3 \text{ A-m}^2 \text{ T}^{-1}$$

18. $\frac{M}{B_H}$ (found in the previous problem) = 3.75×10^3 A-m^2 T^{-1}

$\theta = 37°$, $\quad d = ?$

$$\frac{M}{B_H} = \frac{4\pi}{\mu_0}(d^2 + \ell^2)^{3/2}\tan\theta$$

$\ell << d$ $\quad$ neglecting ℓ w.r.t.d

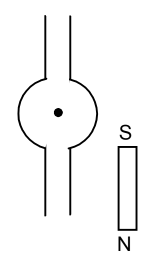
$$\Rightarrow \frac{M}{B_H} = \frac{4\pi}{\mu_0} d^3 \text{Tan}\theta \Rightarrow 3.75 \times 10^3 = \frac{1}{10^{-7}} \times d^3 \times 0.75$$

$$\Rightarrow d^3 = \frac{3.75 \times 10^3 \times 10^{-7}}{0.75} = 5 \times 10^{-4}$$

$\Rightarrow d = 0.079$ m $= 7.9$ cm

19. Given $\frac{M}{B_H} = 40$ A-m^2/T

Since the magnet is short 'ℓ' can be neglected



$$\text{So, } \frac{M}{B_H} = \frac{4\pi}{\mu_0} \times \frac{d^3}{2} = 40$$

$$\Rightarrow d^3 = \frac{40 \times 4\pi \times 10^{-7} \times 2}{4\pi} = 8 \times 10^{-6}$$

$\Rightarrow d = 2 \times 10^{-2}$ m $= 2$ cm

with the northpole pointing towards south.

20. According to oscillation magnetometer,

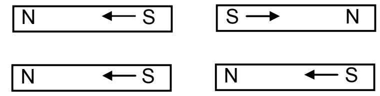
$$T = 2\pi\sqrt{\frac{I}{MB_H}}$$

$$\Rightarrow \frac{\pi}{10} = 2\pi\sqrt{\frac{1.2 \times 10^{-4}}{M \times 30 \times 10^{-6}}}$$

$$\Rightarrow \left(\frac{1}{20}\right)^2 = \frac{1.2 \times 10^{-4}}{M \times 30 \times 10^{-6}}$$

$$\Rightarrow M = \frac{1.2 \times 10^{-4} \times 400}{30 \times 10^{-6}} = 16 \times 10^2 \text{ A-m}^2 = 1600 \text{ A-m}^2$$

21. We know : $\upsilon = \dfrac{1}{2\pi}\sqrt{\dfrac{mB_H}{I}}$



For like poles tied together

$M = M_1 - M_2$

For unlike poles $M' = M_1 + M_2$

$$\frac{\upsilon_1}{\upsilon_2} = \sqrt{\frac{M_1 - M_2}{M_1 + M_2}} \Rightarrow \left(\frac{10}{2}\right)^2 = \frac{M_1 - M_2}{M_1 + M_2} \Rightarrow 25 = \frac{M_1 - M_2}{M_1 + M_2}$$

$$\Rightarrow \frac{26}{24} = \frac{2M_1}{2M_2} \Rightarrow \frac{M_1}{M_2} = \frac{13}{12}$$

22. $B_H = 24 \times 10^{-6}$ T $\qquad T_1 = 0.1'$

$$B = B_H - B_{wire} = 2.4 \times 10^{-6} - \frac{\mu_o}{2\pi}\frac{i}{r} = 24 \times 10^{-6} - \frac{2 \times 10^{-7} \times 18}{0.2} = (24 - 10) \times 10^{-6} = 14 \times 10^{-6}$$

$$T = 2\pi\sqrt{\frac{I}{MB_H}} \qquad \frac{T_1}{T_2} = \sqrt{\frac{B}{B_H}}$$

$$\Rightarrow \frac{0.1}{T_2} = \sqrt{\frac{14 \times 10^{-6}}{24 \times 10^{-6}}} \Rightarrow \left(\frac{0.1}{T_2}\right)^2 = \frac{14}{24} \Rightarrow T_2^{\,2} = \frac{0.01 \times 14}{24} \Rightarrow T_2 = 0.076$$

23. $T = 2\pi\sqrt{\dfrac{I}{MB_H}}$ $\qquad$ Here $I' = 2I$

$T_1 = \dfrac{1}{40}$ min $\qquad T_2 = ?$

$$\frac{T_1}{T_2} = \sqrt{\frac{I}{I'}}$$

$$\Rightarrow \frac{1}{40T_2} = \sqrt{\frac{1}{2}} \Rightarrow \frac{1}{1600T_2^{\,2}} = \frac{1}{2} \Rightarrow T_2^{\,2} = \frac{1}{800} \Rightarrow T_2 = 0.03536 \text{ min}$$

For 1 oscillation Time taken = 0.03536 min.

For 40 Oscillation Time = 4 × 0.03536 = 1.414 = $\sqrt{2}$ min

24. γ_1 = 40 oscillations/minute

$B_H = 25\ \mu T$

m of second magnet = 1.6 A-m^2

d = 20 cm = 0.2 m

(a) For north facing north

$$\gamma_1 = \frac{1}{2\pi}\sqrt{\frac{MB_H}{I}} \qquad \gamma_2 = \frac{1}{2\pi}\sqrt{\frac{M(B_H - B)}{I}}$$

$$B = \frac{\mu_0}{4\pi}\frac{m}{d^3} = \frac{10^{-7} \times 1.6}{8 \times 10^{-3}} = 20\ \mu T$$

$$\frac{\gamma_1}{\gamma_2} = \sqrt{\frac{B}{B_H - B}} \Rightarrow \frac{40}{\gamma_2} = \sqrt{\frac{25}{5}} \Rightarrow \gamma_2 = \frac{40}{\sqrt{5}} = 17.88 \approx 18 \text{ osci/min}$$

(b) For north pole facing south

$$\gamma_1 = \frac{1}{2\pi}\sqrt{\frac{MB_H}{I}} \qquad \gamma_2 = \frac{1}{2\pi}\sqrt{\frac{M(B_H - B)}{I}}$$

$$\frac{\gamma_1}{\gamma_2} = \sqrt{\frac{B}{B_H - B}} \Rightarrow \frac{40}{\gamma_2} = \sqrt{\frac{25}{45}} \Rightarrow \gamma_2 = \frac{40}{\sqrt{\left(\frac{25}{45}\right)}} = 53.66 \approx 54 \text{ osci/min}$$

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